رگرسیون: (Regression)

$x \longrightarrow y$

$$Y = f(x) \longrightarrow \begin{cases} y = x^2 + 136 + e^{x^2} \\ y = \frac{\sqrt{x}}{3} + 3\sqrt[3]{x} \\ y = e^{x-1} \\ y = 2x + 4 \end{cases} \Longrightarrow \text{Historical data}$$

رگرسیون خطی:

$$\boxed{Y = B_0 + B_1 X} \quad \begin{bmatrix} \mu_{y|x} = B_0 + B_1 x \\ E(Y|x) = B_0 + B_1 x \end{bmatrix}$$

$$\longrightarrow y = B_0 + B_1 X + \varepsilon$$

$\varepsilon \longrightarrow$ هدف در رگرسیون به ε min $\rightsquigarrow \text{Min} \sum \varepsilon_i^2 = \ldots\ldots$

$$\xrightarrow{\text{بعد از حل}} \begin{cases} \hat{B}_1 = \dfrac{\sum x_i y_i - \dfrac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \dfrac{(\sum x_i)^2}{n}} \Longrightarrow \text{تخمین شیب خط براساس داده های تاریخی} \\ \hat{B}_0 = \bar{y} - \hat{B}_1 \bar{x} \Longrightarrow \text{تخمین عرض از مبدأ خط براساس داده های تاریخی} \end{cases}$$

$$\left(\hat{B}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \right)$$

$$\sum (x_i - \bar{x})^2 \longrightarrow \sum x_i^2 - n\bar{x}^2$$

$$\begin{cases} SS_E = \sum (y_i - \hat{y}_i)^2 \\ SS_T = \sum (y_i - \bar{y})^2 \end{cases} \Longrightarrow SS_E = SS_T - \hat{B}_1 S_{xy}$$

$$c_i = \frac{x_i - \bar{x}}{\sum (x_i - \bar{x})^2} \rightsquigarrow \sum c_i = 0 \rightsquigarrow \sum c_i^2 = \frac{1}{S_{xx}} \qquad \hat{B}_1 = \sum c_i y_i$$

$$d_i = \frac{1}{n} - \bar{x} c_i \rightsquigarrow \sum d_i = 1 \rightsquigarrow \sum d_i^2 = \frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \qquad \hat{B}_0 = \sum d_i y_i$$

$$\sum c_i d_i = -\frac{\bar{x}}{S_{xx}}$$

نتایج:

$$E(\hat{B}_0)=B_0$$

$$E(\hat{B}_1)=B_1$$

$$Var(\hat{B}_0)=\sigma^2\left(\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right)$$

$$Var(\hat{B}_1)=\frac{\sigma^2}{S_{xx}}$$

$$\left(\hat{\sigma}^2=\frac{SS_E}{n-2}\right)$$

چرا به دست می‌آوریم؟؟

① $$T_{n-2}=\frac{\hat{B}_1-B_{10}}{\frac{\hat{\sigma}}{\sqrt{S_{xx}}}} \longrightarrow H_0=?$$

② $$T'_{n-2}=\frac{\hat{B}_0-B_{00}}{\hat{\sigma}\sqrt{\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}}} \longrightarrow H_0=?$$

$$CI_{①}:\left[\hat{B}_1-t_{\frac{\alpha}{2},n-2}\cdot\frac{\hat{\sigma}}{\sqrt{S_{xx}}}\ ,\ \hat{B}_1+t_{\frac{\alpha}{2},n-2}\frac{\hat{\sigma}}{\sqrt{S_{xx}}}\right]$$

$$CI_{②}:\left[\hat{B}_0-t_{\frac{\alpha}{2},n-2}\cdot\hat{\sigma}\sqrt{\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}}\ ,\ \hat{B}_0+t_{\frac{\alpha}{2},n-2}\hat{\sigma}\sqrt{\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}}\right]$$

$$AR_{①}:\left[-t_{\frac{\alpha}{2},n-2}\ \ t_{\frac{\alpha}{2},n-2}\right] \qquad AR_{②}:\left[-t_{\frac{\alpha}{2},n-2}\ ,\ t_{\frac{\alpha}{2},n-2}\right]$$

برآورد فاصله‌ای برای متغیر y به ازای مقدار داده شده از متغیر مستقل x:

آماره: $$\frac{y_0-\hat{y}_0-0}{\sqrt{\hat{\sigma}^2\left(1+\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}}\right)}}\sim t_{n-2}$$

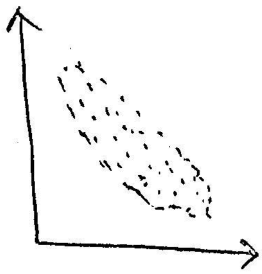
$$\longrightarrow \hat{y}_0-t_{\frac{\alpha}{2},n-2}\sqrt{\cdots}\leq y_0\leq \hat{y}_0+t_{\frac{\alpha}{2},n-2}\sqrt{\cdots}$$

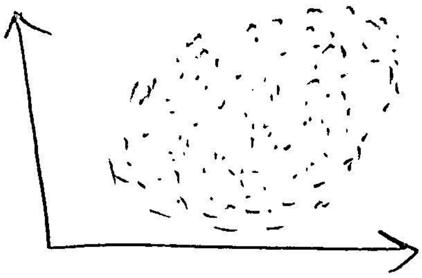
برآورد فاصله‌ای برای میانگین مقدار متغیر پاسخ به ازای مقدار داده شده از متغیر مستقل:

آماره: $$\frac{\hat{\mu}_{y|x_0}-\mu_{y|x_0}}{\sqrt{\hat{\sigma}^2\left(\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}}\right)}}$$

$$\longrightarrow \hat{\mu}_{y|x_0}-t_{\frac{\alpha}{2},n-2}\sqrt{\cdots}\leq\mu_{y|x_0}\leq\hat{\mu}_{y|x_0}+t_{\frac{\alpha}{2},n-2}\sqrt{\cdots}$$

همبستگی : Correlation





$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{Cov(x,y)}{\sqrt{Var(x)\, Var(y)}} \xrightarrow{\text{تخمین}} r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

$$= \hat{\beta}_1 \sqrt{\frac{S_{xx}}{S_{yy}}}$$

$$\begin{cases} H_0: & \rho = 0 \\ H_1: & \rho \neq 0 \end{cases}$$

آماره : $|t|_{n-2} = \left| \frac{r}{\sqrt{1-r^2}} \right| \sqrt{n-2}$

$$AR = [-t_{\frac{\alpha}{2}, n-2} , t_{\frac{\alpha}{2}, n-2}] \longrightarrow X \ ?$$

یک سؤال ! : اگر این آزمون رو بلد نباشیم، چطوری می‌تونیم همبستگی یا عدم همبستگی رو مشخص کنیم؟

Subject:
Year: Month: Date:

سوال یک تمرین: الف)

$$\begin{cases} S_{xx} = 0.68 \\ S_{xy} = 10.17 \end{cases} \qquad \begin{cases} \bar{x} = 1.196 \\ \bar{y} = 92.16 \end{cases}$$

$$\hat{\sigma}^2 = \frac{SS_E}{n-2} = \frac{\sum (y_i - \hat{y}_i)^2}{n-2} = 1.18$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = 14.95$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 74.28$$

معادله خط رگرسیون:

$$y = 14.95x + 74.28$$

ب)

$$\hat{\beta}_1 - t_{0.025,18} \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} \leq \beta_1 \leq \hat{\beta}_1 + t_{0.025,18} \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$$

$$14.95 - 2.101 \sqrt{\frac{1.18}{0.68}} \leq \beta_1 \leq 14.95 + 2.101 \sqrt{\frac{1.18}{0.68}}$$

$$\hat{\beta}_0 - t_{0.025,18}\, \hat{\sigma} \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \leq \hat{\beta}_0 \leq \hat{\beta}_0 + t_{0.025,18}\, \hat{\sigma} \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}}$$

$$74.28 - 2.101 \times \sqrt{1.18} \sqrt{\ldots} \leq \hat{\beta}_0 \leq 74.28 + 2.101 \times \sqrt{1.18} \sqrt{\ldots}$$

CURRENT

Subject:

Year: Month: Date:

ج)

$H_0: B_1 = 13$

$H_1: B_1 \neq 13$

$$T_{n-2} = \frac{\hat{B}_1 - B_1}{\frac{\hat{\sigma}}{\sqrt{S_{xx}}}} = \frac{14.95 - 13}{\frac{\sqrt{1.18}}{\sqrt{0.68}}} = 1.48$$

$AR = (-2.88 , 2.88) \longrightarrow$ دلیلی برای رد فرض صفر وجود ندارد

د)

$x_0 = 1$

$$\hat{\mu}_{y|x} - t_{\frac{\alpha}{2}, n-2}\sqrt{\cdots\cdots} \leq \mu_{y|x} \leq \hat{\mu}_{y|x} + t_{\frac{\alpha}{2}, n-2}\sqrt{\cdots\cdots}$$

$$\hat{\mu}_{y|x_0} = \hat{B}_0 + \hat{B}_1 x_0 = 74.28 + 14.95(1) = 89.23$$

$$89.23 - 2.101\sqrt{1.18\left(\frac{1}{20} + \cdots\right)} \leq \mu_{y|x} \leq 89.23 + 2.101\sqrt{1.18\left(\frac{1}{20} + \cdots\right.}$$

ه)

$$\hat{y} - t_{\frac{\alpha}{2}, n-2}\sqrt{\hat{\sigma}^2\left(1 + \frac{1}{n} + \cdots\right)} \leq y \leq \hat{y} + t_{\frac{\alpha}{2}, n-2}\sqrt{\hat{\sigma}^2\left(1 + \frac{1}{n} + \cdots\right.}$$

$$86.63 \leq y \leq 91.63$$

2) الف)

$\bar{x} = 182.42$	$\bar{y} = 145.62$
$\sum x = 4743$	$\sum y = 37.86$
$\sum x^2 = 880545$	$\sum y^2 = 555802$

$n = 26$ $\sum x_i y_i = 697076$

CURRENT

$S_{xx} = \sum (x_i - \bar{x})^2 = 15312.35$

$S_{xy} = 6432.23$

$\rightsquigarrow \hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = 69.10$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 0.42$

$\longrightarrow y = 69.10x + 0.42$

ب)

$\begin{cases} H_0: \rho = 0 \\ H_1: \rho \neq 0 \end{cases}$

$r = \hat{\beta}_1 \sqrt{\frac{S_{xx}}{S_{yy}}} = 0.77$

$|t| = \left| \frac{r}{\sqrt{1-r^2}} \right| \sqrt{n-2} = 5.91 \qquad AR = [0 \quad 2.046]$

فرض صفر رد می‌شود.