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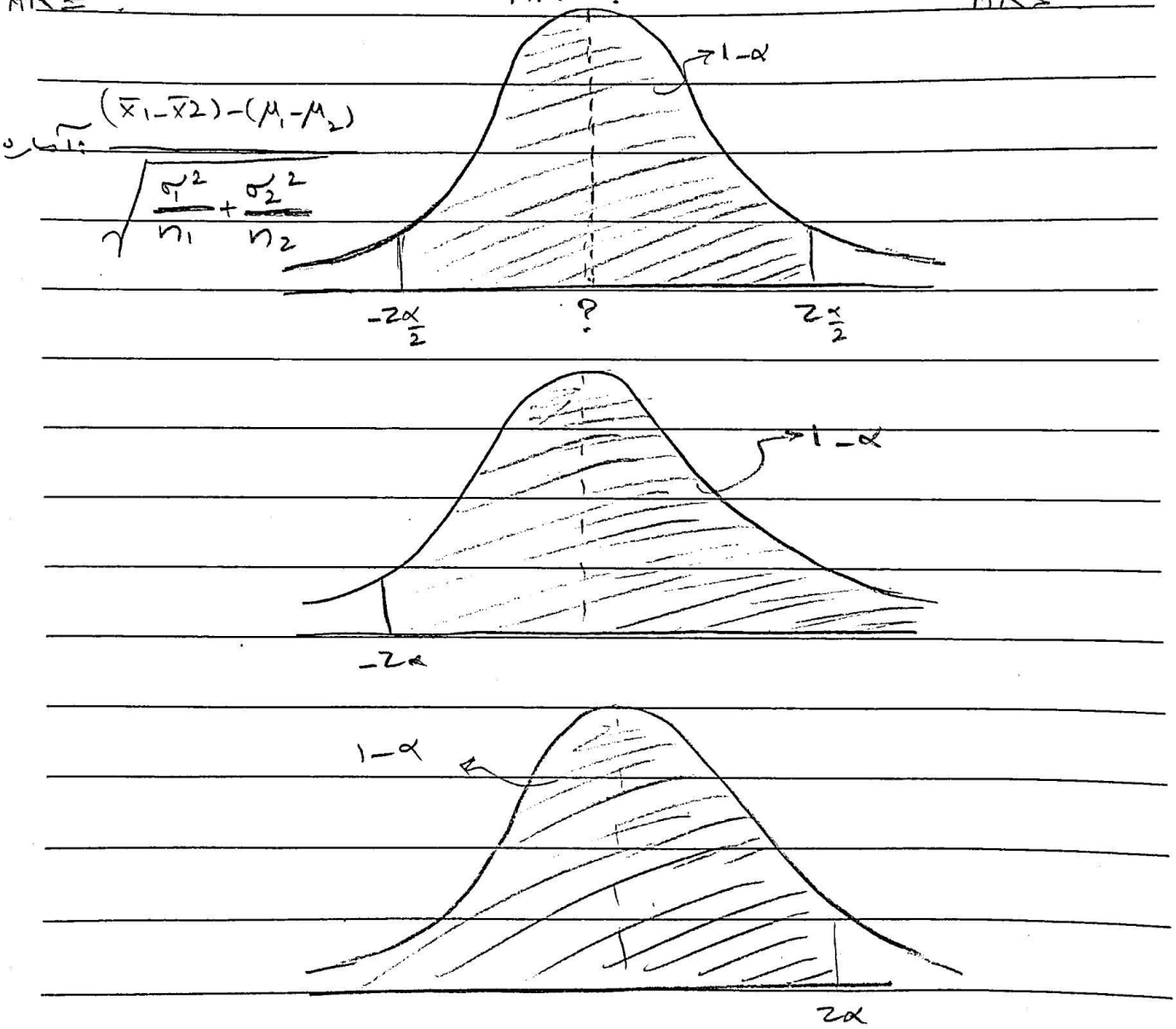
آزمون فرض 2 پارامتری:

حالت اول: آزمون فرض اختلاف میانگین زمانی σ_2 ها مشخص است:

$H_0: \mu_1 - \mu_2 = \Delta$ | $H_0: \mu_1 - \mu_2 \leq \Delta$ | $H_0: \mu_1 - \mu_2 \geq \Delta$
$H_1: \mu_1 - \mu_2 \neq \Delta$ | $H_1: \mu_1 - \mu_2 > \Delta$ | $H_1: \mu_1 - \mu_2 < \Delta$

AR = ? | AR = ? | AR = ?



حجم نمونه برای برآورد میانگین با یک احتمال مشخص:

$$n \geq \frac{(z_{\frac{\alpha}{2}} + z_\beta)^2 (\sigma_1^2 + \sigma_2^2)}{\Delta^2} \quad (n_1 = n_2 = n)$$

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ب) اگر $\sigma_1^2 = \sigma_2^2$ ولی نامعلوم باشد

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$H_0: \mu_1 - \mu_2 = \Delta$	$H_0: \mu_1 - \mu_2 \leq \Delta$	$H_0: \mu_1 - \mu_2 \geq \Delta$
$H_1: \mu_1 - \mu_2 \neq \Delta$	$H_1: \mu_1 - \mu_2 > \Delta$	$H_1: \mu_1 - \mu_2 < \Delta$
$AR = [-t_{\frac{\alpha}{2},\upsilon} < t < t_{\frac{\alpha}{2},\upsilon}]$	$AR = ?$	$AR = ?$

ج) اگر نامعلوم و نابرابر

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - \Delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \longrightarrow \upsilon = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$H_0: \mu_1 - \mu_2 = \Delta$	$H_0: \mu_1 - \mu_2 \leq \Delta$	$H_0: \mu_1 - \mu_2 \geq \Delta$
$H_1: \mu_1 - \mu_2 \neq \Delta$	$H_1: \mu_1 - \mu_2 > \Delta$	$H_1: \mu_1 - \mu_2 < \Delta$
$AR = [-t_{\frac{\alpha}{2},\upsilon} < t < t_{\frac{\alpha}{2},\upsilon}]$	$AR = (-\infty < t_{\alpha,\upsilon}]$	$AR = [-t_{\alpha,\upsilon} < \infty)$

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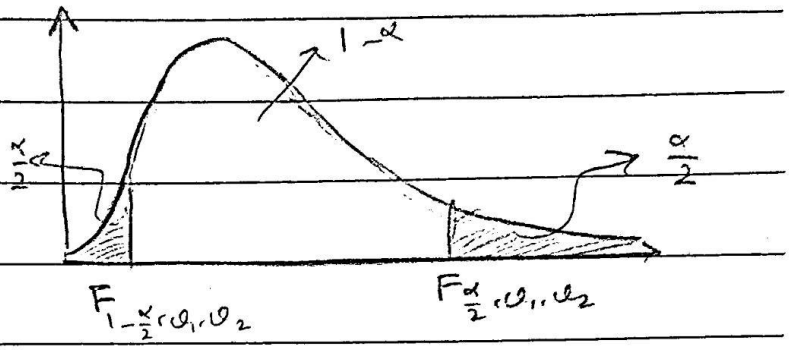
آزمون فرض نسبت واریانس 2 جامعه:

$$F=\left(\frac{S_1}{S_2}\right)^2\left(\frac{\sigma_2}{\sigma_1}\right)^2$$



$H_0: \sigma_1^2=\sigma_2^2$	$H_0: \sigma_1^2=\sigma_2^2$	$H_0: \sigma_1^2=\sigma_2^2$
$H_1: \sigma_1^2\neq\sigma_2^2$	$H_1: \sigma_1^2<\sigma_2^2$	$H_1: \sigma_1^2>\sigma_2^2$
	$AR=?$	$AR=?$

$$AR=\left[F_{1-\frac{\alpha}{2},n_1-1,n_2-1} \;,\; F_{\frac{\alpha}{2},n_1-1,n_2-1}\right]$$

آزمون فرض برای تفاضل نسبت 2 جامعه:

$$Z_0=\frac{(\hat{P}_1-\hat{P}_2)-(P_1-P_2)}{\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1}+\frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}}$$

$\hat{P}$ مشترک

$$\hat{P}=\frac{x_1+x_2}{n_1+n_2}$$

$$(\hat{P}_1-\hat{P}_2)-\Delta$$

$$Z_0=\frac{(\hat{P}_1-\hat{P}_2)-\Delta}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$H_0: P_1-P_2=\Delta$	$H_0: P_1-P_2=\Delta$	$H_0: P_1-P_2=\Delta$
$H_1: P_1-P_2\neq\Delta$	$H_1: P_1-P_2<\Delta$	$H_1: P_1-P_2<\Delta$
$AR=?$	$AR=?$	$AR=?$

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آزمون t زوجی یا Paired-t test : (σ مجهول)

(آزمون میان μ 2 جامعه است که عبارتست)

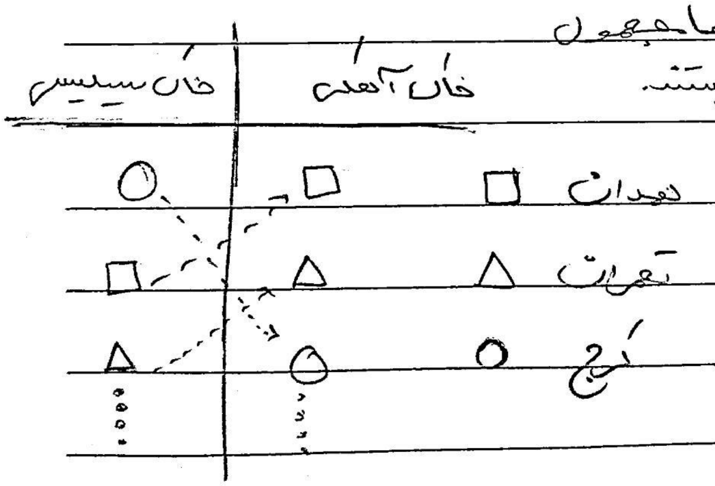
σ جوامع مشخص نیست.

دیتاها به صورت زوج زوج هستند.

یعنی جای دیتاها رو نمی‌تونیم عوض کنیم!

* ظاهر سؤال به صورت حالت دوم اما دیتاها قابل جابجایی نیستند.

(آزمون فرض μ 2 جامعه وقتی σها مجهول و برابر نیستند.)



$d_i = x_i - y_i$

$H_0: \mu_D = 0$

$H_1: \mu_D \neq 0$

$t_0 = \frac{\bar{d} - 0}{\frac{S_D}{\sqrt{n}}}$

$AR = ?$

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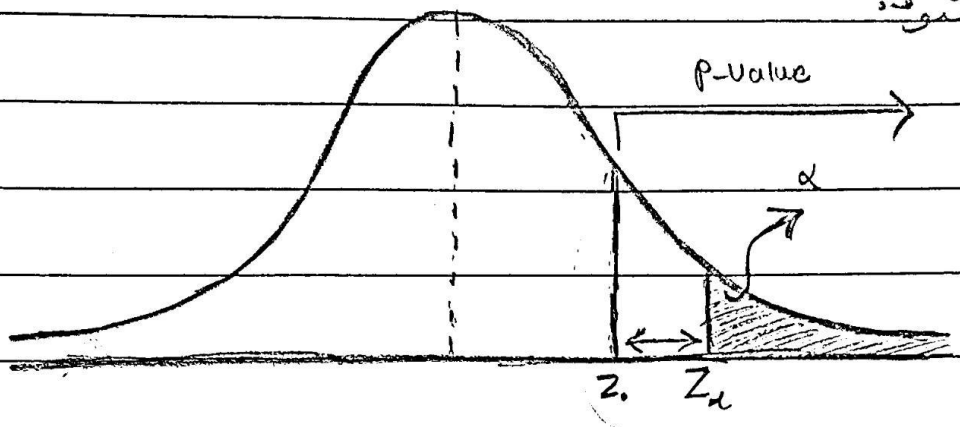
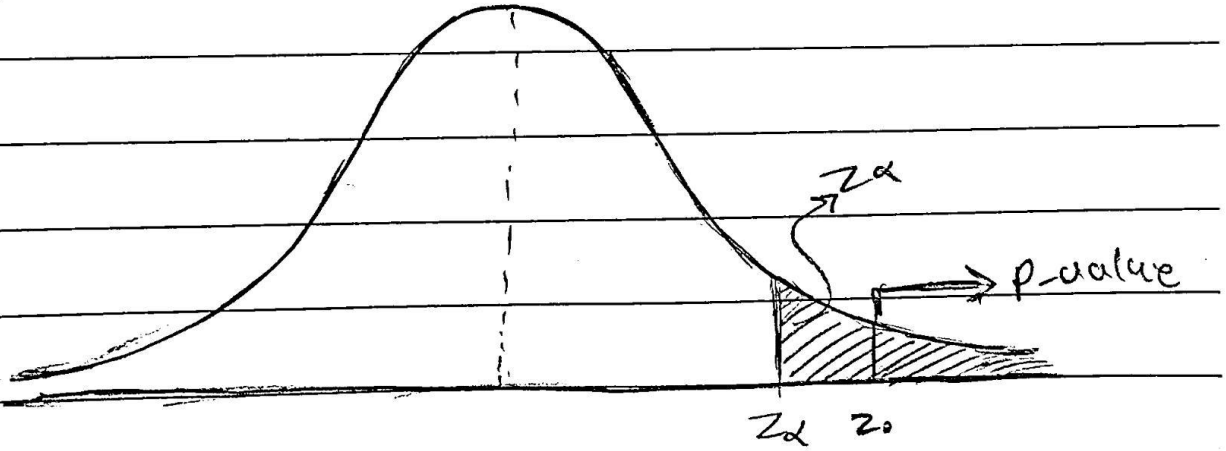
P_value: کوچک‌ترین مقدار α که منجر به رد فرض صفر می‌شود.



$AR = (-\infty, z_\alpha]$ ⟹ $\alpha < p\_value$ H_0 ✓ ✗

P_value: $p(Z > z_0)$

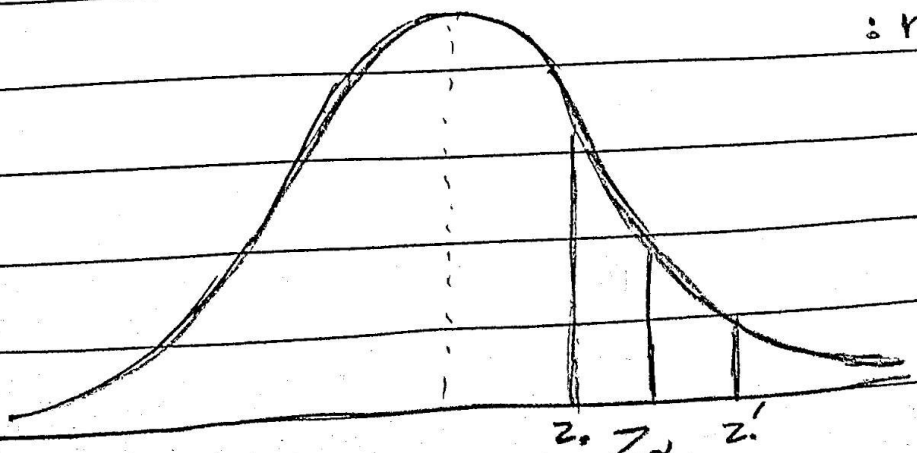


$AR = (-\infty, z_\alpha]$ ⟹ $\alpha > p\_value$ H_0 ✓ ✗

P_value: ?

تأثیر n:

$$z_0 = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



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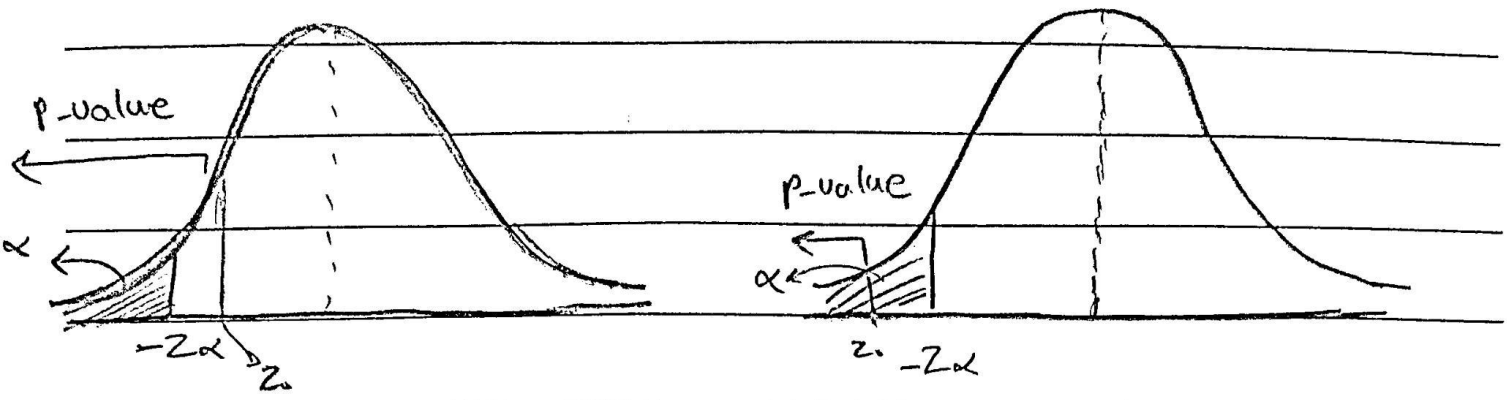
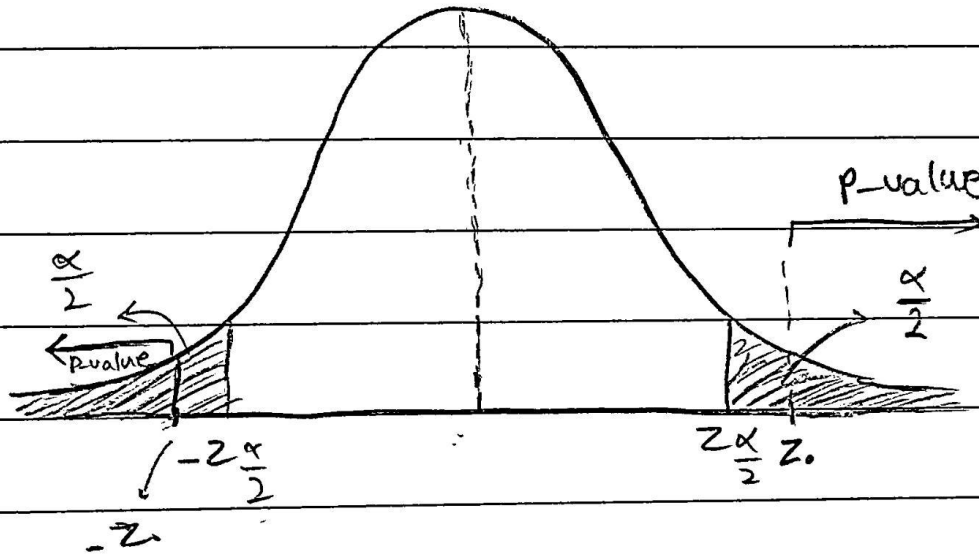


p_value : $p(Z < Z_0)$ p_value : $p(Z < Z_0)$



p_value: $p(Z > Z_0) + p(Z < -Z_0)$

$\Rightarrow 1 - p(Z < Z_0) + p(Z > Z_0)$

$= 1 - p(Z < Z_0) + 1 - p(Z < Z_0)$

$= 2 - 2p(Z < Z_0) = 2(1 - p(Z < Z_0))$

$= 2(1 - \Phi(Z_0))$

(1)

$\begin{cases} \bar{x}_1 = 8.73 \\ s_1^2 = 0.35 \\ n_1 = 15 \end{cases}$ $\begin{cases} \bar{x}_2 = 8.58 \\ s_2^2 = 0.40 \\ n_2 = 17 \end{cases}$ $\alpha = 0.05$

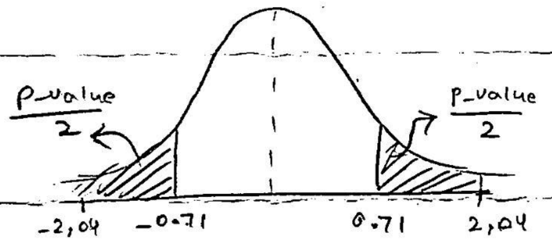
$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{cases}$

$$t_0 = \frac{(8.73) - (8.58) - 0}{\sqrt{\frac{0.35}{15} + \frac{0.40}{17}}} = +0.71$$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 29.88 \simeq 30 \rightarrow AR = (-t_{0.025, 30}, t_{0.025, 30})$$

$$= (-2.042, 2.042)$$

دلیلی بر رد H_0 نداریم.



$\rightsquigarrow \alpha < p\text{-value} \Longrightarrow$ H_0 رد نمی‌شود.

(2) الف) paired t-test $D_i = B_i - A_i \Longrightarrow \bar{D} = \frac{\sum D_i}{n}$, $s_D^2 = \frac{\sum (D_i - \bar{D})^2}{n-1}$

$\begin{cases} H_0: \mu_D = 0 \\ H_1: \mu_D \neq 0 \end{cases}$

$$\bar{D} = 17 , S_D = 6.41 \rightsquigarrow d_0 = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = \frac{17 - 0}{\frac{6.41}{\sqrt{10}}} = 8.38$$

$$AR = (-t_{\frac{\alpha}{2}, \nu}, t_{\frac{\alpha}{2}, \nu}) = (-2.26, 2.26) \longrightarrow$$ فرض صفر رد می‌شود

ب)

$\begin{cases} \mu_D = 10 \\ \mu_D \neq 10 \end{cases} \longrightarrow d_0 = 3.45$ $AR = (-2.26, 2.26) \longrightarrow$ فرض صفر رد می‌شود

ج)

$1 - \beta = 0.9 \rightsquigarrow \beta = 0.1$ $d = \frac{10 - 0}{6.41} = 1.56 \xrightarrow{OC} n \simeq 8$

بجای S

پس $n = 10$ اعلیم!

(3)

$$\begin{cases} H_0: \frac{\sigma_1}{\sigma_2}=1 \Rightarrow \sigma_1=\sigma_2 \\ H_1: \frac{\sigma_1}{\sigma_2}\neq 1 \Rightarrow \sigma_1\neq\sigma_2 \end{cases} \qquad \alpha=0.1$$

$$F_0=\left(\frac{S_1}{S_2}\right)^2\left(\frac{\sigma_2}{\sigma_1}\right)^2=\frac{2.3}{1.9}=1.21$$

$$AR=\left[F_{0.95,14,14} \quad F_{0.05,14,14}\right]=\left[\frac{1}{2.47} \quad 2.47\right]=[0.40 \quad 2.47]$$

دلیلی بر رد H_0 نداریم.

(4)

$$\begin{cases} n_1=500 \\ \hat{p}_1=\frac{385}{500}=0.77 \end{cases} \qquad \begin{cases} n_2=400 \\ \hat{p}_2=\frac{267}{400}=0.66 \end{cases} \qquad \alpha=0.05$$

$$\begin{cases} H_0: p_1=p_2 \\ H_1: p_1\neq p_2 \end{cases} \qquad \hat{p}=\frac{267+385}{500+400}=0.72$$

$$Z_0=\frac{\hat{p}_1-\hat{p}_2-0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}=3.32$$

$AR=[-1.96 \quad 1.96]$ ⟶ H_0 رد می‌شود.