P_value | تمرین ها | پیام ها

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برآورد فاصله‌ای (دو پارامتری)

$\boxed{\text{توضیح اولیه}}$

< تفاوت با تک پارامتری؟ چه روشی؟؟

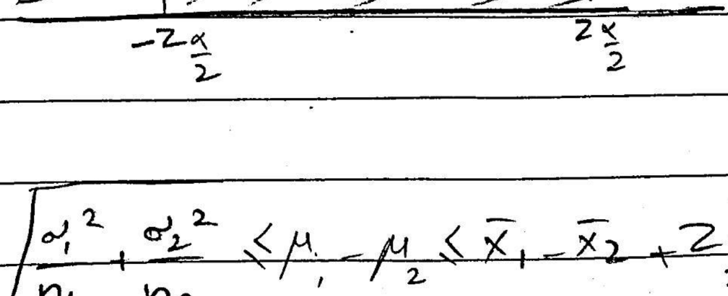
الف) برآورد اختلاف میانگین ها زمانی که σ^2 مشخص باشد.

دوطرفه: $$P\left(-Z_{\frac{\alpha}{2}} < \frac{\bar{X}_1-\bar{X}_2-\mu_1-\mu_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} < Z_{\frac{\alpha}{2}}\right) = 1-\alpha$$

حاشیه

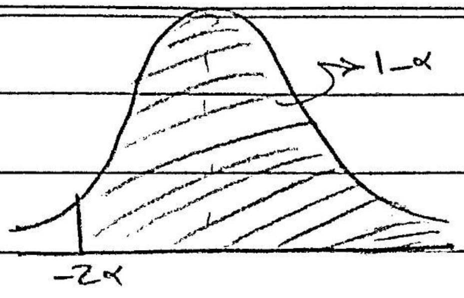
$$X_1 \sim N(\mu_1, \sigma_1^2) \xRightarrow{\text{احتمال}} X_1 - X_2 \sim N(\mu_1-\mu_2, \sigma_1^2+\sigma_2^2)$$
$$X_2 \sim N(\mu_2, \sigma_2^2)$$

$$\bar{X}_1 \sim N\left(\mu_1, \frac{\sigma_1^2}{n}\right)$$
$$\bar{X}_2 \sim N\left(\mu_2, \frac{\sigma_2^2}{n}\right) \Longrightarrow \bar{X}_1-\bar{X}_2 \sim N\left(\mu_1+\mu_2, \frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right)$$



$$\bar{X}_1-\bar{X}_2-Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} < \mu_1-\mu_2 < \bar{X}_1-\bar{X}_2+Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

یک‌طرفه:



$$P\left(\frac{\bar{X}_1-\bar{X}_2-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \geq -Z_\alpha\right)=1-\alpha$$

$$\Rightarrow \boxed{\mu_1-\mu_2 \leq \bar{X}_1-\bar{X}_2+Z_\alpha\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

به همین ترتیب:

$$\boxed{\mu_1-\mu_2 \geq \bar{X}_1-\bar{X}_2-Z_\alpha\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

ب) برآورد فاصله‌ای برای اختلاف میانگین‌ها زمانی که σ_1^2 و σ_2^2 مشخص نباشند اما برابر باشند $(\sigma_1^2=\sigma_2^2=\sigma^2)$

$$\bar{X}_1 \sim N\left(\mu_1, \frac{\sigma^2}{n_1}\right)$$

$$\Rightarrow \bar{X}_1-\bar{X}_2 \sim N\left(\mu_1-\mu_2, \sigma^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\right)$$

$$\bar{X}_2 \sim N\left(\mu_2, \frac{\sigma^2}{n_2}\right)$$

X_1 از جمعیت ۱ $\Rightarrow \frac{(n_1-1)S_x^2}{\sigma^2}$

$$\Rightarrow \frac{(n_1-1)S_x^2}{\sigma^2}+\frac{(n_2-1)S_y^2}{\sigma^2} \sim \chi^2_{(n_1+n_2-2)}$$

X_2 از جمعیت ۲ $\Rightarrow \frac{(n_2-1)S_y^2}{\sigma^2}$

$$t=\frac{\square}{\sqrt{\frac{\bigcirc}{\triangle}}}$$

توزیع t چطور می‌ساختیم؟

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$$\frac{\bar{x}_1-\bar{x}_2-\mu_1-\mu_2}{\sqrt{\sigma^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \Longrightarrow$$ نرمال استاندارد

$$\sqrt{\frac{\frac{(n_1-1)S_x^2}{\sigma^2}+\frac{(n_2-1)S_y^2}{\sigma^2}}{n_1+n_2-2}}$$

$\Longrightarrow \chi^2$ کای 2

$\Longrightarrow$ درجه آزادی

$$\frac{\bar{x}_1-\bar{x}_2-\mu_1-\mu_2}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\sqrt{\frac{(n_1-1)S_x^2+(n_2-1)S_y^2}{n_1+n_2-2}}} \sim t_{n_1+n_2-2}$$

χ^2 شرط نرمال بودن داره ؟

2طرفه: $P(-t_{\frac{\alpha}{2},v} <$ آماره $< t_{\frac{\alpha}{2},v}) = 1-\alpha$

$$\boxed{\bar{x}_1-\bar{x}_2-S_p t_{\frac{\alpha}{2},v}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} < \mu_1-\mu_2 < \bar{x}_1+\bar{x}_2+S_p t_{\frac{\alpha}{2},v}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

یک طرفه: $P($ آماره $\geq -Z\alpha) = 1-\alpha$

$$\boxed{\mu_1-\mu_2 < \bar{x}_1-\bar{x}_2+t_{\alpha,v}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$P($ آماره $< Z\alpha) = 1-\alpha$

$$\boxed{\mu_1-\mu_2 \geq \bar{x}_1-\bar{x}_2-t_{\alpha,v}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

ج) برآورد میانگین وقتی σ_1^2 و σ_2^2 مجهول و برابر نیستند

$$T_v = \frac{\bar{X}_1 - \bar{X}_2 - \mu_1 - \mu_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2-1}} = \frac{(a+b)^2}{\frac{a^2}{n_1-1} + \frac{b^2}{n_2-1}}$$

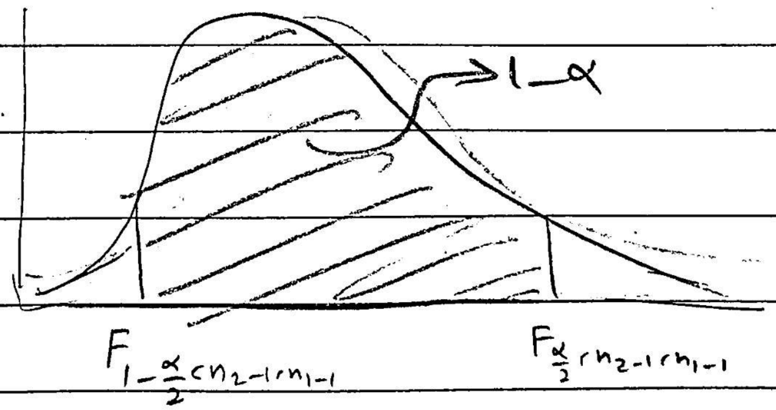
$$\boxed{\bar{X}_1 - \bar{X}_2 - t_{\frac{\alpha}{2}, v}\sqrt{\dots} < \mu_1 - \mu_2 < \bar{X}_1 + \bar{X}_2 + t_{\frac{\alpha}{2}, v}\sqrt{\dots}}$$

مقایسه واریانس دو جامعه:

$$\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{(n-1)}$$

$$\frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{(n-1)}$$

$$\Longrightarrow \frac{\frac{(n_2-1)S_2^2}{(n_2-1)\sigma_2^2}}{\frac{(n_1-1)S_1^2}{(n_1-1)\sigma_1^2}} = \left(\frac{S_2}{S_1}\right)^2 \left(\frac{\sigma_1}{\sigma_2}\right)^2$$



$$p\left(F_{1-\frac{\alpha}{2}, n_2-1, n_1-1} < \left(\frac{S_2}{S_1}\right)^2 \left(\frac{\sigma_1}{\sigma_2}\right)^2 < F_{\frac{\alpha}{2}, n_2-1, n_1-1}\right)$$

$$\boxed{\left(\frac{S_1}{S_2}\right)^2 \times \frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}} < \left(\frac{\sigma_1}{\sigma_2}\right)^2 < \left(\frac{S_1}{S_2}\right)^2 F_{\frac{\alpha}{2}, n_2-1, n_1-1}}$$

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برآورد فاصله‌ای (نسبت) p

$X_1 \sim N(np_1, n_1p_1(1-p_1)) \to \hat{p}_1 = \frac{x_1}{n}$

$\to \hat{p}_1 \sim N\left(p_1, \frac{p_1(1-p_1)}{n_1}\right)$

$X_2 \sim N(np_2, n_2p_2(1-p_2)) \to \hat{p}_2 = \frac{x_2}{n}$

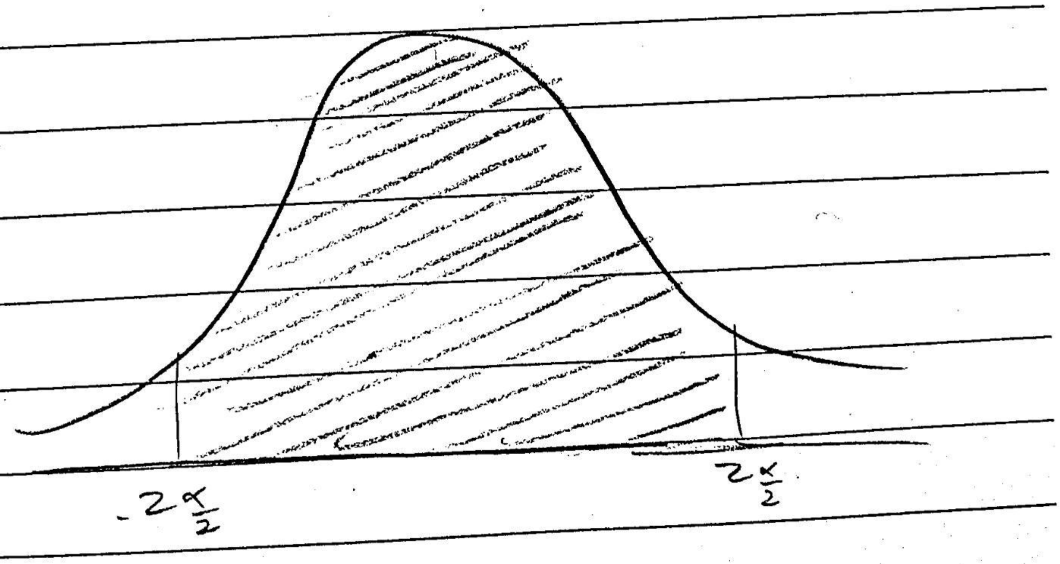
$\to \hat{p}_2 \sim N\left(p_2, \frac{p_2(1-p_2)}{n_2}\right)$

$\to \hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$

$$\to P\left(-Z_{\frac{\alpha}{2}} < \frac{\hat{p}_1 - \hat{p}_2 - p_1 - p_2}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} < Z_{\frac{\alpha}{2}}\right) = 1-\alpha$$

$$\to \hat{p}_1 - \hat{p}_2 - Z_{\frac{\alpha}{2}}\sqrt{\ldots\ldots} < p_1 - p_2 < \hat{p}_1 - \hat{p}_2 + Z_{\frac{\alpha}{2}}\sqrt{\ldots\ldots}$$

توجه: در رادیکال مقادیر (p_2 از $\hat{p}_2$) استفاده شود.



مثال: در کارخانه‌ای از 2 دستگاه ...

① ماشین ①: $\begin{cases} \bar{x}_1 = \frac{\sum x_i}{n_1} = 16.02 \\ \sigma_1 = 0.020 \\ n_1 = 10 \end{cases}$

$\Longrightarrow$ حالت ؟

ماشین ②: $\begin{cases} \bar{x}_2 = \frac{\sum x_i}{n_2} = 16.01 \\ \sigma_2 = 0.025 \\ n_2 = 10 \end{cases}$

$$(L, U) = \left(\bar{x}_1 - \bar{x}_2 - Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},\ \bar{x}_1 - \bar{x}_2 + Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$L: 16.02 - 16.01 - 1.96\sqrt{\frac{(0.020)^2}{10} + \frac{(0.025)^2}{10}} = \ldots$$

$$U: 16.02 - 16.01 + 1.96\sqrt{\frac{(0.020)^2}{10} + \frac{(0.025)^2}{10}} = \ldots$$

② دو تأمین کننده چرخ دنده مورد استفاده ...

چرخ دنده 1: $\begin{cases} \bar{x}_1 = 290 \\ S_1 = 12 \\ n_1 = 10 \end{cases}$ چرخ دنده 2: $\begin{cases} \bar{x}_2 = 321 \\ S_2 = 22 \\ n_2 = 16 \end{cases}$

$$(L, U) = \left(\bar{x}_1 - \bar{x}_2 - S_p t_{\frac{\alpha}{2}, v}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ \bar{x}_1 - \bar{x}_2 + t_{\frac{\alpha}{2}, v} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{9 \times (12)^2 + 15 \times (22)^2}{24} \Rightarrow \boxed{S_p = 18.88}$$

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$$L: 290-321-18.88\times 2.064\times\sqrt{\frac{1}{10}+\frac{1}{16}}$$

$$U: 290-321+18.88\times 2.064\times\sqrt{\frac{1}{10}+\frac{1}{16}}$$

$$\begin{cases} S_{men}=0.98 \\ S_{women}=1.02 \end{cases} \qquad \begin{cases} n_{men}=25 \\ n_{women}=21 \end{cases} \qquad \boxed{1-\alpha=0.98}\ 3$$

$$\left(\frac{S_1}{S_2}\right)^2\times\frac{1}{F_{\frac{\alpha}{2},n_1-1,n_2-1}}<\left(\frac{\sigma_1}{\sigma_2}\right)^2<\left(\frac{S_1}{S_2}\right)^2 F_{\frac{\alpha}{2},n_2-1,n_1-1}$$

$$\frac{S_1}{S_2}\sqrt{\frac{1}{F_{\frac{\alpha}{2},n_1-1,n_2-1}}}<\frac{\sigma_1}{\sigma_2}<\frac{S_1}{S_2}\sqrt{F_{\frac{\alpha}{2},n_2-1,n_1-1}}$$

$$\frac{0.98}{1.02}\sqrt{\frac{1}{\cdots}}<\frac{\sigma_1}{\sigma_2}<\frac{0.98}{1.02}\sqrt{\cdots}$$

باید از روی جدول چی رو بخونیم؟

$F_{0.01,24,20}$ و $F_{0.01,20,24}$

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$$\begin{cases} n_1 = 300 \\ \text{تعداد معیوب} : 15 \end{cases} \qquad \begin{cases} n_2 = 300 \\ \text{تعداد معیوب} : 8 \end{cases} \qquad \alpha = 0.05 \quad 4$$

$$\hat{p}_1 = \frac{15}{300} \qquad \hat{p}_2 = \frac{8}{300}$$

$$(1-\alpha)\ \hat{p}_1 - \hat{p}_2 - Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$L = \frac{15}{300} - \frac{8}{300} - 1.96 \sqrt{\frac{\frac{15}{300}\left(1-\frac{15}{300}\right)}{300} + \frac{\left(\frac{8}{300}\right)\left(1-\frac{8}{300}\right)}{300}} = \cdots$$

$$U = \hat{p}_1 - \hat{p}_2 + Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \cdots$$

$$U = \frac{15}{300} - \frac{8}{300} + 1.96 \sqrt{\frac{\frac{15}{300}\left(1-\frac{15}{300}\right)}{300} + \frac{\frac{8}{300}\left(1-\frac{8}{300}\right)}{300}} = \cdots$$