نمونه سئوالات (برآورد فاصله‌ای)

① $X \sim N(\mu , \sigma^2) \longrightarrow \bar{X} \sim N(\mu , \frac{\sigma^2}{n}) \Rightarrow \tilde{X} \sim N(\mu , \frac{(0.005)^2}{n})$

$|\mu - \bar{x}| < 0.002 \Longrightarrow -0.002 < \mu - \bar{x} < \underbrace{0.002}_{Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}}$

$\Longrightarrow Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 0.002$

$\Longrightarrow n = \frac{(1.96)^2 \times (0.005)^2}{(0.002)^2} = 25$

② (الف)

$$\begin{cases} \bar{x} = 177 \\ \sigma = 5 \\ Z_{\frac{\alpha}{2}} = 2.5758 \end{cases} \Longrightarrow (L , U) = (\bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} , \bar{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$$

$\Longrightarrow (L , U) = (170.50 , 183.44)$

ب) $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = 17.33 \Longrightarrow s = 4.163$

$\longrightarrow (L , U) = (\bar{x} - t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} , \bar{x} + t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}})$

$\longrightarrow (L , U) = (171.64 , 182.361)$

③ $\sum (x_i - \bar{x})^2 = 17.8 \Longrightarrow s^2 = \frac{17.8}{15-1} = 1.27 \Longrightarrow s = 1.12$

$\chi^2_{0.025, 14} = 26.12$, $\chi^2_{1-0.025, 14} = 5.63$

Subject:

Year. Month. Date. ( )

$\longrightarrow (L , U) = \left(1.127\sqrt{\frac{14}{26.12}} , 1.127\sqrt{\frac{14}{5.53}}\right) \longrightarrow$ 2طرفه

یکطرفه بالایی : $(0 , U) = \left(0 , 1.127\sqrt{\frac{14}{\boxed{6.57}}}\right) \longrightarrow \chi^2_{1-\alpha}$

④ الف)

$1-\alpha = 0.95 \Longrightarrow Z_{\frac{\alpha}{2}} = 1.96$

$\hat{p} = \frac{x}{n} = \frac{18}{50} = \frac{9}{25}$

$\longrightarrow \hat{p} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\frac{9}{25} - 1.96\sqrt{\frac{\frac{9}{25}(1-\frac{9}{25})}{50}} < p < \frac{9}{25} + 1.96\sqrt{\frac{\frac{9}{25}(1-\frac{9}{25})}{50}}$

ب)

$1-\alpha = /99$

$Z_{\frac{\alpha}{2}} = 2.58$

$E = |\hat{p} - p| \leq Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\Longrightarrow E \qquad \xrightarrow{2} Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < 0.02$

$\Longrightarrow 2.58\sqrt{\frac{\frac{9}{25}(1-\frac{9}{25})}{n}} < 0.02$

$\Longrightarrow (2.58)^2 \times \frac{0.36 \times 0.64}{n} < (0.02)^2 \Longrightarrow n > 3804$

2000

Subject:

Year. Month. Date. ( )

توجه: می‌توانستیم به جای $\hat{p} = \frac{9}{25}$ در قسمت ب) سؤال قبل از $\text{Max}\ \hat{p}(1-\hat{p}) = \frac{1}{4}$ استفاده کنیم. یعنی:

$$\Rightarrow Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < 0.02 \Rightarrow 2.58 \times \sqrt{\frac{\frac{1}{4}}{n}} < 0.02$$

$$\Rightarrow n > \frac{1}{4}\left(\frac{Z_{\frac{\alpha}{2}}}{0.02}\right)^2 \Rightarrow n > \ldots$$

این عبارت یعنی اگر برآوردی از $\hat{p}$ نداشته باشیم می‌توانیم در بدترین حالت از $\text{Max}\ \hat{p}(1-\hat{p}) = \frac{1}{4}$ استفاده کنیم